Bridging the Gap between Face to Face and Online Maths Tutoring.

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Abstract

These are the preliminary findings of a study of mathematical dialogues in e-learning. It attempts to identify the key communication strategies used when learning and discussing mathematics in face to face environments and map these onto a set of requirements for collaborative areas in Learning Managements Systems (LMS). The current lack of features for mathematical collaboration in LMSs is effectively disenfranchising students and staff in mathematically rich subject areas at a time when recruiting for these subjects is increasingly difficult. Collaboration, of both a formal and an informal nature, is part of the learning process; this is increasingly taking place on-line in many subjects.

Whilst a rise in student numbers has led to the majority of UK higher education institutions investing in some form of Learning Management System, an overall drop in the numbers of students studying mathematics after the age of 17 has been seen in the UK. In parallel with this, the mathematical knowledge of students entering mathematics, science and engineering, is perceived to be declining.

Using an Activity Theory methodology, video data was collected and a preliminary analysis of these determined the major categories of communications strategy. The key strategies identified included the verbal and written language of mathematics, gestures, sharing multiple documents and behaviour modelling. An investigation of some web based learning environments was also made and the features compared with these emerging categories.

In current Learning Management systems, the only features provided for mathematical dialogues involve menu driven equation creation tools in areas with virtual whiteboards and synchronous chat.

Other existing web-based mathematical tools include some of the features identified as requirements. But none addresses the need for speed and ease of use in synchronous chat areas, which are widely used by students who are still learning the subject and do not have the skills of experts. Although some existing tools can point the way to better facilities in the LMSs some new features need to be created to address the remaining requirements.

Introduction

These are the preliminary findings of research into mathematical dialogues in e-learning systems. The aim of this research project is to develop a set of requirements for communications software that will inform the development of software for students and staff working collaboratively on mathematically rich texts.

There are shortcomings with the communications areas, such as the asynchronous bulletin boards and the synchronous chat/messaging, in Learning Management Systems (LMS) when they are required for mathematically rich discussions. This paper seeks to identify a list of requirements for such areas by looking at how students and staff behave in traditional tutorials and how these behaviours can be translated into appropriate software tools. It highlights the difficulties with, and shortcomings of, existing systems, whilst suggesting existing models that could be envisaged as part of a more unified scheme.

Data has been collected on the nature of communications areas in the learning management system Blackboard® and a number of alternative tools for mathematical communications, including a web-based bulletin board used for mathematical material by school children. In addition, new video material of mathematics tutorials for physics and engineering postgraduates in a face to face environment has been used to investigate the strategies for communicating mathematical information and solving problems used by staff and students. By comparing features of the on-line environment with how students and academics communicate face to face, a picture is emerging of the disenfranchisement of mathematically rich subject areas by the introduction of LMS/VLE[1]. This project brings together research in education, learning technology and Usability (Human Computer Interaction) under the single methodology of Activity Theory.

Context

There are plans to increase participation in UK universities to 50% of school leavers by 2010 with no commitment to provide additional funding for these increasing numbers. In view of the larger numbers of students, alternative modes of delivery are inevitable. Professor Mark Stiles states in his article "Staying on Track - Why are we using technology in teaching? (Stiles, 2004)

> "The reality for many students is lectures taken alongside two or three hundred and sometimes more of their peers, packed tutorials and staff who, despite being conscientious and willing, find it difficult to give time to the individual who needs their help."

In order to address these problems, UK Universities have invested heavily in LMSs. Over 85% of UK Higher Education Institutions in a 2003 survey have installed a VLE/LMS (Browne and Jenkins, 2003).

Many universities have adopted the proprietary Learning Management Systems such as Blackboard® and WebCT® to deliver blended learning and e-learning to their students. However, these packages were clearly not designed with mathematics and science in mind. Although some efforts have been made to bolt on equation editing tools, especially a version of WebEQ™ from Design Science Inc., these were not available in

[1] LMS= Learning Management System, VLE = Virtual Learning Environment., These terms are used interchangeably for a type of web based system that provides an integrated platform for staff to distribute notes, create e-learning materials, on-line testing, assignments, announcements and for students to view course details, learning materials, upload assignments, post messages, chat and perform other administrative and course related tasks. Each course module has its own area and these systems can synchronised with a university's registry and library databases.

earlier versions of the LMS products, implying the developers had not planned for them at the outset. They are either not available or not very easy to use in the key communications areas that allow students to collaborate to solve problems or ask questions of tutors. Therefore, if these systems are to be made fit for learning mathematics, it is essential that a well researched set of requirements be provided to developers. Similar problems inevitably exist in the field of distance learning, although as a more mature area, teaching materials are designed specifically for self-directed study.

Mathematical skills amongst UK undergraduates entering higher education are thought to be declining: one of the major issues of concern addressed by UK government's *The Post–14 Mathematics Inquiry* (Smith, 2004) was:

> "The failure of the current curriculum, assessment and qualifications framework in England, Wales and Northern Ireland to meet the needs of many learners and to satisfy the requirements and expectations of employers and higher education institutions"

To help address these issues, mathematics should not be marginalised with special tools that "only mathematicians use". Mathematical tools should be part of the mainstream provision; after all, not only mathematicians but most scientists and engineers, technologists, economists and psychologists and many other disciplines use mathematics directly or indirectly. Everybody should be able to see and use the features so that they are familiar and do not identify mathematical dialogues as in some ways special or difficult.

Data Collection

a) Face to Face tutorials

A week long series of lectures and tutorials were video recorded at a mathematics summer school for postgraduate acousticians funded by the UK "Engineering and Physical Sciences Research Council" (EPSRC). These were then analysed by looking for both communications strategies by lecturers and, where the tutors and students are working with others, as well as learning strategies, which form the supporting environment that they create for themselves within the constraints of the tutorial rooms and lecture rooms. The lectures were interspersed by tutorials with smaller groups which combined "mini-lecture" style, where a tutor solved a problem on the backboard (in the traditional sense!) and more individual style, where students worked on a series of problems, asking questions and working with their peers or the tutors if they wished.

It should be noted that, unlike the normal university situation, the summer school course was intensive, but there were no assessments. The students, who had been sent by their supervisors, were drawn from a range of UK institutions and had not met together as a group prior to this course. Most students had a background in numerate subjects. These classes addressed the specific mathematical needs of those studying acoustics, which bridges physics and engineering. The lecturers and additional tutors were also drawn from a range of institutions.

Most of the lectures and short sections of the tutorials were recorded. Two digital video cameras were used. Audio from the lectures was also captured separately using a mini-disc recorder.

b) Interfaces in Learning Management Systems.

The interface of the Blackboard® LMS version 6 was investigated. Previous work on interfaces (Leventhall, 2002) showed that current mathematical equation creation/editing tools (such as in word processors) had menu driven interfaces and those users surveyed did not like them, finding these slow and awkward to use for large amounts of material. They showed a preference for more natural language tools as were available in some earlier products. It also showed that some mathematics lecturers were not familiar with the alternatives such as LaTeX.

Also investigated were alternative web based tools which allow collaborative mathematically rich work, where equations are displayed immediately. For comparison, a standard text bulletin board (with no special maths tools available) which was used for posting mathematically rich material was included in the study. This was included as there were too few mathematical postings available for analysis on the local Blackboard® system.

Analysis

The Methodology

Activity Theory is used to guide and inform the whole research project of which this paper is a segment. It brings together the two main disciplines involved - Human Computer Interaction (HCI)/Usability and education.

From an educational perspective, the context of learning is an essential element in this study. The focus is on students' and tutors' collaboration being mediated by tools (the LMS), their objectives and the rules they negotiate to work by, as well as the institutional context.

Used in educational settings, from early childhood (Bodrova and Leong, 1996) to the adult environment such as work in organizational learning (Engeström et al., 1999). Activity Theory encompasses concepts now part of mainstream educational practise such as the Zone of Proximal Development.

Activity theory is also starting to be used in the HCI and Usability disciplines to research the requirements for, and analyse the usability of, new computer software. These areas had previously been influenced by the disciplines of ergonomics and cognitive psychology with less attention paid to other more social aspects of software use. Nardi (1996) suggests that the traditional HCI approaches have not provided users with software that truly reflect all their requirements and a new approach is required. The idea that a tool is an important source of socialization put forward by Kaptelinin (1996) suggests that alternatives to more traditional research and development methods are required. Kuutti (1999) states that "new approaches must be established in order to master change, and since Activity Theory was developed precisely to study individual and social transformation, it is a strong candidate for forming the background for such new approaches."

Features of the interfaces in Learning Management Systems.

In version 5 of Blackboard®, no integral tool was available for entering equations. In version 6 however, a pop-up equation tool[2] based on the WebEQ tool from Design Science Inc. was integrated into several areas and one of these was the Virtual Classroom, where it was available for use in both the Chat area and the Virtual Whiteboard.

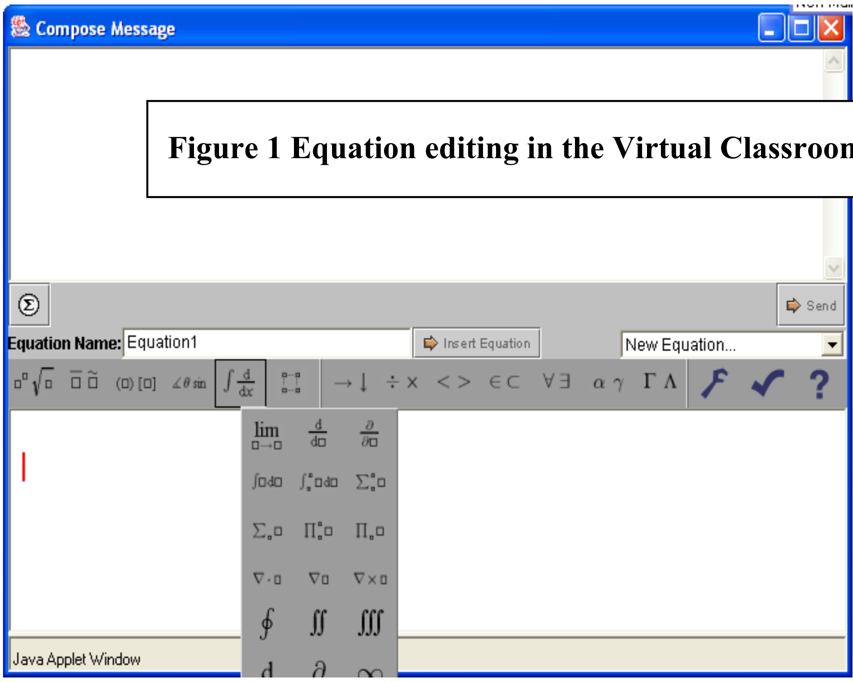

Figure 1 Equation editing in the Virtual Classroom Chat area

In the web based Blackboard® learning environments, the main communication areas where students collaborate are the asynchronous bulletin board area, called a discussion board, and in the synchronous “chat". Email is also available.

In the Virtual Classroom, the chat area and the Virtual Whiteboard, provide an equation tool as a pop-up menu allowing the input of equations, shown in Figure 1. The equation is displayed in the top window once the user clicks on the ⇒Send button. To input text and an equation the user has to move between the two areas of the window, typing any text in the top part and using the Σ button to pop up a window below where the equation is created, then clicking on the ⇒Insert Equation button to incorporate it. All this is time-consuming as each symbol must be picked from drop down menus, which often need to be searched through to find the correct symbol. A novice user would not necessarily know where to look. Compare this with the desire to send messages quickly, as evidenced by the speed of SMS texting and the use of Microsoft Messenger with its own acronyms and emoticons to cut down typing. It would suggest that a menu driven system might be perceived as too slow to be usable.

The chat area is only a small segment of the screen as can be seen in Figure 2

In the Office Hours area of the Virtual Classroom, there is an additional “lightweight Chat” which does not include the equation editing tool.

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[2] There is an inherent problem with pop-up tools, if a user has a pop-up blocking feature on their computer (such as on the Google toolbar) to block unwanted advertisements and distasteful material, it can also block the pop-up equation editing tool.

The asynchronous bulletin board-style discussion area provides for text, HTML or Smart Tags for input, but there was no explicit means to input or to display equations properly. In the bulletin board discussion area, the only means of sharing

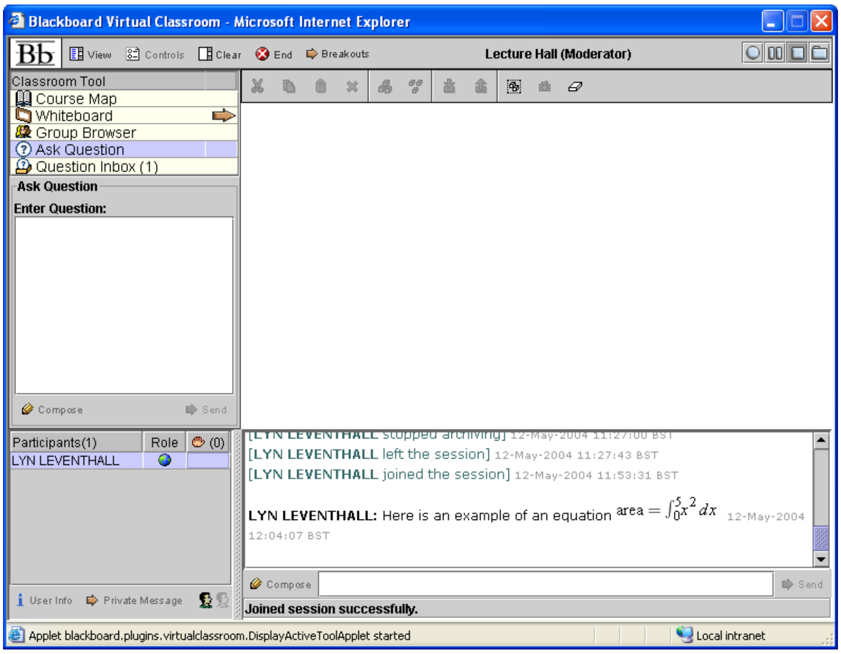

Figure 2 Virtual Classroom showing Chat area with equation

mathematically rich material was by uploading a document to the bulletin board discussion area. This meant that, for word-processed documents or documents created using a mathematical software package e.g. MathCad, the person reading the file had to be using the same (or compatible) software as the author. Few students would have a specialist PDF authoring package, although most have a PDF reader.

Once sent, of course it is not possible to change or edit anything posted in the chat area, although this is not true for the virtual whiteboard.

Discussion of the Blackboard® tools

The web based e-learning system described above addresses only the primary need to produce mathematical equations on-screen in collaborative areas; they do not give additional ways of communicating. Neither is consideration given to the way students use chat areas for quick messages, where shortened forms and acronyms are essential elements. If written texts were adequate on their own, why go beyond giving students a text book and expecting them to teach themselves? Most students need much more than this, they need the interaction that comes from a "live" teacher and peer group, they need to be part of a Community of Practice (Wenger, 1998)[3], centred on their subject, where they share a common purpose. Part of this sharing, means being able to communicate with the community and be inducted into its ways by existing members.

[3] This may be explored in another paper.

Analysis of the Face to Face tutorials

The video material was analysed using a simple content analysis to look for strategies employed by tutors and students when discussing mathematical material – solving problems or explaining concepts. From this, the elements that emerged point to the following items as being widely used strategies for learning mathematical concepts.

The Language of Mathematics

To teach the language of mathematics, we need verbal /spoken descriptions as well as written. The notation is shorthand for the spoken word. Clearly, to communicate with others, the mathematical language must be learnt. It is apparent from the video data that some teachers articulate the equation whilst writing it on the blackboard. Others read it aloud, after writing it. Some omit this stage, especially if they are using prewritten material, either multimedia displays (e.g. PowerPoint slides) or even handwritten acetates. The written and spoken language should go together as they are both part of the same communications process.

During a lecture, an experienced mathematician will often write down a familiar equation from left to right reading/saying it aloud it at the same time. This will not be the same order in which a similar equation is produced when trying to solve a particular problem. Although the outline shape may be put in place, many of the elements are added "out of order".

During a left to right rendering of the equation, the language will be used to help the students learn how to read the mathematics they see written. During the problem solving rendering, the students will see real mathematical behaviour modelled. The order of writing can be crucial when problem solving, as this models the way in which the solution is built up. Pre-prepared materials do not show this order. Neither does the use of the equation editing tool in BlackBoard® which produces an equation "fully formed". This will tell the students nothing about the thought processes going to form this equation or expression. Neither can they see the relationships between elements and, say, a graph that can be pointed out as the building of an equation/expression progresses.

Gestures

Both the lecturers and the students used two distinct types of gestures: pointing and illustrative. Pointing indicates something, whilst the illustrative gestures gave more information about an object, such as the motion of a sound wave or the three dimensional nature of an entity. It has been shown (Roth and Welzel, 2001) that when learning new material and discussing mathematical concepts, gestures are an important part. Research in mathematics education confirms that gestures help children learn (Alibali et al., 1999) Teachers also use gesture (Alibali et al., 1997)

A current project at the University of Chicago (McNeill, 2004)focuses on creating avatars (cartoon like animated images) with correct gestures, for use in distance learning. Their web page describes the background to this project:

> "In a highly spatial and contextually rich interaction, providing a sense of situatedness between the tutor and student is crucial in facilitating learning. This situatedness relies on temporal as well as spatial dimensions. Spatial situatedness is obviously essential for deictic references, such as references to elements in the interlocutor's space and references to artefacts of the interaction (e.g. graphical illustrations of the tutorial content). Maintaining this situatedness

> is especially critical in disciplines where space, diagrams and graphical representations are of great importance (e.g. engineering and mathematics).
>
> "... we believe that these elements of normal discourse are essential in assisting the student to grasp the substance of the tutorial..."

It would appear that the above project is essentially dealing with creating web based applications for delivering tutorials and explanatory material.

Students also gesture when discussing new material, as they have not mastered all the elements of the new language constructs (Roth and Lawless, 2002, Goldin-Meadow et al., 1999). They also need to tie ideas to elements on the page, demonstrate special ideas and motion and indicate relationships.

Use of Multiple Documents

Most of the students used multiple documents when working on solutions to problems. They arranged and rearranged their own notes, the lecturer's handouts, model answers etc. to refer to when required. It would suggest that they were linking the ideas in these documents to their own solutions. For students working on-line, this linking of their solution to other texts is possible (assuming the other texts are also on-line) by opening multiple windows.

Sharing

When working together, students were sharing, discussing and pointing to the multiple documents as well as their own solutions and written ideas. This behaviour was observed in all the tutorial situations. They write on each others' work or point and compare their solutions with those of their peers. Tutors, moderating these discussions in this face to face situation, also point to students' work and the other documents as well as writing on either a separate sheet or the students' own work.

Modelling Behaviour - How to be a mathematician

During tutorials, some lecturers solved a given problem on a blackboard for the students to see. They did not always have a copy of the solution so had to work through the solution for themselves, in the same way that the students might. This was modelling the way of solving the problem with a thinking aloud element that helped the students to understand the lecturer's approach to the problem. As several lecturers/tutors were present in some tutorials, there was also discussion between them about the way to solve the problems and they worked on problems on the blackboard much as they would in their offices.

One role of the tutor is to model the behaviour of a mathematician / engineer. Bligh (1998) suggests little work has been done on behaviour modelling and a recent literature search suggests that this is still the case. Whilst solving complex problems on a blackboard for a group of seated students, the tutor demonstrates the "how" part of problem solving. Students see that it is easy to make mistakes and this is part of the learning process. At more advanced levels, lecturers go through iterations of the solution, making false starts and resolving inconsistencies. This is not apparent with pre-written material as the answer is often presented as a finished product. There are no hints of how it was really arrived at, the false starts and the messy process of refining your answer. Another element that the tutor demonstrates is the reuse of elements: by rubbing out part of an expression and using it as the template for a similar problem. This automatically links a series of solutions together. During the process of solving a problem, the tutor verbalises the ideas and thoughts behind the strategy. He/she demonstrates how to lay out the work, which is not usually in the neat form found in the pages of a book or on the web page, by placing elements around the board for reference as well as making little sketches and aide-memoire notes.

Zone of Proximal Development

It was observed that a tutor helped an individual student to achieve the solution to a problem that otherwise would have been just beyond the student's capabilities when working alone. This is described as the Zone of Proximal development (ZPD) (Vygotsky and Cole, 1978). Although this is widely used as a teaching strategy with school children, it was happening quite naturally in these post graduate classes as well.

Written work

All the students' work was handwritten and most of the lecturers hand wrote on the blackboards and on paper during the tutorials. During the lectures, there was a mixture: pre-prepared OHP slides that were handwritten, OHP slides that were printed, direct use of the blackboards using chalk, pre-prepared multimedia slides created with both LaTeX and Microsoft PowerPoint. Most of the lecturers' notes were printed; no information is available about how these were created.

An essential part of the learning process is the range of behaviours used by students and their tutors. Together they negotiate the meaning of mathematics and the solution of problems with gestures, sharing, multiple documents, behaviour modelling and linking the verbal and written language of mathematics.

Summary of the communication strategies used face to face tutorials

The following is a list of the broad categories of communication and problem solving strategies:

- The use of hand-written symbols and hand drawn graphs alongside pre-prepared materials.
- Talk in the language of mathematics supplemented by gestures and pointing.
- Gestures indicating motion or spatial relationships to the text, graphs, diagrams and sketches. (e.g. elaboration of the 3-D nature of a 2-D diagram)
- Whilst helping each other to solve problems, students often share their solutions and they write on or annotate the solution of their collaborators.
- Sharing and use multiple documents, surrounding themselves with copies of notes, model solutions and reference books, the tutors handouts, work sheets etc. They refer to these as they are formulating their own solutions.
- Writing equations and saying them aloud, either simultaneously or just afterwards.
- Whilst working with mathematical software in a computer suite, the students also used the pointing, gesturing and sharing behaviours using the computer screen as "virtual paper". This was a most interesting, but not unexpected result.
- Use of the Zone of Proximal Development (ZPD)

These behaviours are not mutually exclusive and are used in combination, tutors also add to the complexity by observing the situation, responding to questions and pointing to errors or other material.

What is missing from the LMS

Although it was possible to input and display an equation, in order to discuss it, the user had to be able to express their thoughts coherently and unambiguously enough for others to respond. This is difficult when learning new material. As shown by (Roth and Lawless, 2002), gestures are used when new concepts and language are being acquired simultaneously. It is a vital part of the learning process. The following list gives a partial picture of the difficulties in using an LMS to teach, discuss and collaborate on mathematically rich texts.

- It is difficult to show the building up of a single equation if an equation editing tool is used and the result displayed "fully formed" on the Whiteboard or the Chat area.
- There is no means of annotating the work of others, except by uploading and downloading files in the bulletin board area; there is no other obvious way of sharing a document where the whole document can be viewed and edited by several parties..
- It is difficult to view and share multiple documents, such as notes, in the virtual classroom.
- In order to acquire the language of mathematics, the verbal element is missing and there is no provision in the virtual classroom for sound clips.
- A facility is needed for showing gestures both pointing and 3D.
- There is no facility for sound in these areas to hear the equations spoken.

Requirements for a mathematically enhanced LMS system

From the analysis, it is clear that the existing communication tools are not adequate for teaching and learning to take place in mathematical rich subjects.

An ideal system could contain some of the following:

1. Integrated equation building tool, that is not simply a "pop-up" window menu driven style, with a simplified LaTeX or natural language style input and an SMS/ messenger set of agreed shortcuts for speed when the user is familiar with them. The mathematical elements should be a normal part of the display and a hidden tool reserved for specialists.
2. Drawing tools that are appropriate for mathematical sketches – with "quick" axes that can be dragged in if required. There could be a customisable drag and drop elements.
3. Multiple documents, viewable simultaneously, with means of highlighting, pointing to individual elements and linking these to the discussion. With accompanying shortcut "emoticon style" indicators to indicate pointing words like: this, that, here, there - to be included in the dialogue
4. Shared documents editable by at least two people/students in real time.
5. Sound to link written equations to the spoken version.
6. A way of seeing the building of equations/expressions rather than a finished entity.
7. A reusable / re-editable equation in a discussion thread so that elements of a previous post can be quickly cut then edited and posted in the current message to save time.
8. Some way of showing 3-D gestures to indicate position, direction etc. associated with diagrams. Possibly with the use of avatars.

Existing tools that partly fulfil these requirements

There are a range of tools and products that fulfil some of the requirements given above, this list is not exhaustive but indicates what else is available..

[Requirements 1, 2, 3 & 6] A system that allows the construction of equations and the sharing of a document as well as graphics and other mathematical tools is **NetTutor / WWWhiteboard** (from Link Systems http://www.link-systems.com/) It is aimed at the Distance Learning market.

The description of this tool on Link Systems own web site http://www.link-systems.com/ is:

> "NetTutor™ is a revolutionary new distance learning environment for the dissemination of online content. NetTutor™ is a web-based graphical chat, threaded and platform independent, allowing students to use their own computers to access learning materials in a non-linear fashion or in real-time with live corresponding tutors."

There is a large mathematical virtual whiteboard called WWWhiteboard that provides a messaging interface with equation menu and drawing tools.

A project to evaluate this product for distance learning, in the USA, suggests that tutors and students like the system (Smith et al., 2003). However, it makes reference to the fact that the tutors find that it is unfortunate that it is not integrated into a learning management system such as Blackboard® or WebCT®, as both they and the students have to switch between systems when discussing mathematics.

[Requirement 1, 4 & 7] Another type of web based document is the **Wiki.** These documents can be modified freely by anyone, allowing collaborative working. They also support hyperlinks and have simplified ways of creating links on the fly. The software runs on a server.

Several maths-enabled Wikis are now available. Some are open source. They allow input as LaTeX or a simplified version of LaTeX. One example was written by Peter Jipsen of Chapman University (USA), who has developed one that renders "on the fly" (Jipsen, 2004) You can type in the equation and see it appear "like magic" on the page below the text box. This software is available from the developer.

[Requirement 1] **WIMS** http://wims.unice.fr/wims/en_home.html is a purpose written system with more than just bulletin boards; it has pages for maths problems, a graphing facility and a range of other features. It uses a LaTeX style input. The instructions state that the

"Facilities for powerful while easily accessible math rendering Mathematical symbols and formulas can be rendered using very tolerant syntaxes: practically any unambiguously recognizable formula can be rendered correctly."

[Requirement 1] A bulletin boards, where school age children post mathematical questions and solutions, can be used as illustrative examples. A UK site at http://nrich.maths.org/discus/messages/board-topics.html is **NRich**. This is a LaTeX style interface for more able school children and undergraduates, an enrichment project where mathematical problems are posted for interested children to attempt and then post their work. There are also bulletin boards for students in different age groups to post questions, the questions are answered by maths teachers and post graduate students. There is a full description for the user about how to format equations and many posts use some of these features. Although sometimes the posts contain the equation in formatted in a way that indicates that Microsoft Excel or a similar tool is known to the student as the use of * and / for multiply and divide occurs as does the ^ symbol for raise to the power. These equations do not appear in the same form on the page as they would have done had LaTeX been used.

[Requirement 5] Version 2.0 of Design Science's Mathplayer™ Plug-in for Microsoft Internet Explorer now has a command that will speak an equation. This is the first version and has some limitations. However it points the way to future use. It is intended to be linked to screen reading products to improve accessibility for the visually impaired but is also could be used in other contexts.

The Technological Foundations

Various technologies are now available for viewing both mathematically rich material and graphs. These are the XML technologies MathML and SVG. Many users also use Java applets for interactive web pages. There is now no good reason why developers shouldn't incorporate elements using these into LMS systems. They are becoming widely used in other areas.

Additional Requirements

Many students use messaging systems such as Microsoft Messenger® for informal discussions alongside the LMS (Stone et al., 2003). In addition, the use of SMS or "texting" on mobile phones is ubiquitous. Neither of these systems provides for mathematical tools. With the newer generation of mobile phones and a synthesis of PDA technologies and mobile phone technology it will soon be possible to use pen or stylus to create mathematical symbols. A system was described which uses a PocketPC PDA (Wan and Watt, 2002).

Use of cameras on mobile phones and webcams within a Microsoft Messenger® style system can allow for graphs, diagrams and sketches to be displayed, although the pictures are not ideal. Sound can be used here too so a tutor can read an equation aloud.

Graphics tablets can give a user the opportunity to input diagrams and hand drawn symbols but this would currently require separate software. However, these elements could show the way an equation is built up. It is possible to store the strokes and replay them as a video.

Suggestions for future developments of software tools

A system for pointing to the texts, highlighting elements of equations and linking elements from two or more documents would seem an essential part of any mathematics tutoring systems. Whilst accompanying this with an emoticon element or shortcut key to that indicate "this bit", "that bit", "here", "there", "this way" and "that way" will go some way to alleviating the problems of not being able to express themselves fluently in the appropriate language.

A transparent annotation frame or layer could be placed over a shared space such as both a messenger window and an accompanying document (e.g. the problem sheet), so that links could be drawn using the mouse from the discussion text to the document. This would need to be smart enough to move as the discussion progresses, keeping the links in step.

This might not be too difficult for developers given that software such as Microsoft NetMeetings™ with shared whiteboards already exist.

Further research

As these are the preliminary findings, the next stages of the work are:

- To use a qualitative analysis software tool (Atlas-ti) to develop a finer-grained analysis of the video material.
- Use some of the software such as the mathematical Wiki in trials with students.
- Discover the experiences of teachers and tutors with Latex based tools, such as the Nrich bulletin board and form a richer picture of the problems and expectations with these.
- Use Activity Theory to draw together these and other elements to further refine the usability requirements for interfaces for mathematical dialogue tools.

Conclusion

Students and tutors discussing mathematics use a wide variety of verbal and non verbal behaviours to convey information to their peers. These include: gestures, shared writing space and other documents, as well as the language of mathematics both spoken and written. These must be reflected in the communications areas of learning management systems if students are to learn effectively.

The Blackboard® learning management system has yet to provide a useful interface for mathematical discussions. However, many technologies and products already exist that could be adapted to improve the situation. Further developments are within the capabilities of developers as the technologies such as MathML and SVG are already in place. A clear idea of what is required to discuss mathematics needs to be agreed. Future systems should not disenfranchise those who need to use mathematically rich language. All systems designed for educational purposes should have these elements properly embedded in them so as not to imply that mathematics is somehow a special case. Any pop-up tools should be easy and quick to use and not slow down the communication and creation of mathematical expressions. The aim should be to make students fluent in the language of mathematics. This can never be the case if they have to resort to an old-fashioned typesetting style of picking individual symbols from a menu and filling in a template.

What are the next steps? We should be pressing developers to add features for mathematical communications now, possibly forming an interest group, encouraging providers and purchasers of e-learning software to make appropriate mathematical tools a basic requirement of any system specification.

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